THE OHIO STATE UNIVERSITY

BLACK MATH HISTORY
HIDDEN FIGURES REVEALED

Matroid Puzzle Game

Grade or age level: Grades 3-8 **Time:** 50 minutes **Form of work:** Pairs

Background

Dr. Carolyn Mahoney graduated from The Ohio State University in 1983 and studied combinatorics, graph theory, and matroids, which will be explored through a game in the following activity. Dr. Mahoney focused on matroid properties and the graphical representations of matroids. In addition to her work with mathematics, Dr. Mahoney aimed to make education accessible for students in impoverished and primarily non-white communities. Dr. Mahoney strongly advocated for professional development for teachers in these regions to further their understanding of the material, and she also strongly urged mathematicians to help examine and develop math curriculums in schools.

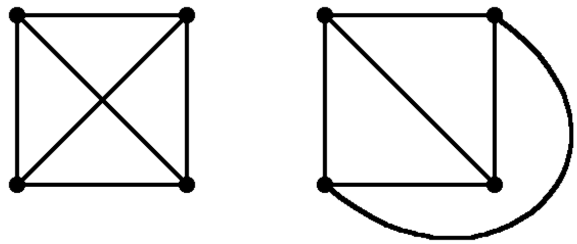
Matroids

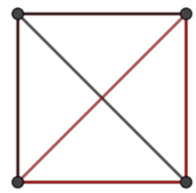
A matroid is a collection of two sets where one set (called the ground set) has a finite, or countable, number of elements and the other set (called the circuits) contains subsets of the first set that satisfy a few properties. The details are beyond the scope of this document, but the interested reader can refer to the references section for further reading.

Matroids have applications in engineering, computer science, geometry, topology, etc. Interestingly, the highest prize in mathematics, the Fields Medal, was recently awarded to Dr. June Huh for his work on matroids. Part of his work was joint with Dr. Eric Katz, a professor at the Department of Mathematics at The Ohio State University.

Graphs

Graphs are another mathematical object. In other fields of study, graphs are sometimes called networks. A graph consists of vertices or nodes and lines (or edges) connecting them. It is important to clarify that not all nodes need to be connected. In fact, some graphs might have isolated vertices that are not connected to anything. What makes a graph that graph is the specific connections between its vertices. For example, the following two graphs are the same even though they are drawn in a slightly different way:

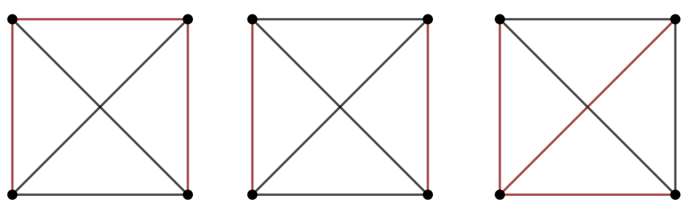




We can think of graphs as paths where we can only walk on edges. On any given graph, a cycle is a path on a graph that takes you back to the starting point without repeating nodes. For example, in the graph above, the edges that form the square make up a cycle. Any of the triangles with vertices on the graph nodes also constitutes a cycle.

Graphic Matroids

There's a specific type of matroids which are related to graphs: graphic matroids. These are the matroids that this activity deals with. Given any graph, the cycles on it make up a matroid: the edges constitute the ground set, while the cycles are the circuits. Moreover, any set of edges that does not contain a cycle is called an independent set, a central concept in matroids. The following are examples of independent sets on the graph from before:



Graphic matroids are also called polygonal matroids, since finding cycles on a graph is very much like finding polygons.

Objectives

- To discover the hidden lives of Black mathematicians from Ohio State.
- To realize that the valuable work and stories of some people remain hidden and we have the power to rectify this.
- To identify patterns in polygon formation.
- To introduce students to graphs and cycles on graphs.

Links with Standards

SOCIAL SCIENCES	
Grade	Standard
2	4. Biographies can show how peoples' actions have shaped the world in which we live.
4	8. Many technological innovations that originated in Ohio benefitted the United States
MATHEMATICS	
Grade	Standard
2, 3	Reason with shapes and their attributes.
4, 5	Classify two-dimensional figures into categories based on their properties.
7	Draw, construct, and describe geometrical figures and describe the relationships between them.
Practice 1	Make sense of problems and persevere in solving them.
Practice 2	Reason abstractly and quantitatively.
Practice 3	Construct viable arguments and critique the reasoning of others.
Practice 8	Look for and express regularity in repeated reasoning.

Materials

Students will work in pairs. Each pair needs the following:

- At least one game board. Boards are included at the end of this document.
- Two different colored dry erase markers. Depending on your setting, pencils, crayons, highlights, could also work. See note below.

Note: Students will be marking on the boards and will play several rounds. To avoid wasting paper, we recommend laminating the boards or inserting them into plastic sleeves. That way, using dry erase markers they can play, then erase and play again.

Opening and Motivation (20 minutes)

1. Introduce Dr. Mahoney to students using materials gathered from the Hidden Figures Revealed project.
2. Introduce the activity by mentioning that it is related to her PhD work. Briefly explain that she worked with something called "matroids." Some of the matroids she studied arise in

networks where we have points connected by some lines. In mathematics, these are called "graphs." Show an example and ask students to propose other examples.

3. Next, explain that when working with matroids, it is important to identify cycles in the graph. Again, show a few examples of what we mean by a cycle and get the students to notice that this is like identifying polygons on the graph.

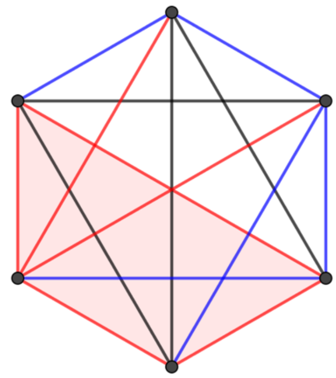
Game Rules

In this activity, students will be playing a two-player game that can be played on different boards (any graph makes up a board). The game is in a way opposite to dots-and-boxes, since the goal is not to form a polygon (cycle).

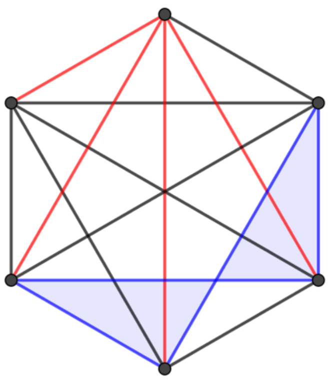
Through this game, students will get familiar with cycles on graphs and groups of edges that contain no cycles (i.e. independent sets on a graphic matroid).

- Each player gets a color marker.
- Players should decide who gets to start and take turns after that.
- On each turn, the player must claim an edge on the board by highlighting it with their color.
- Only previously unclaimed (black) edges can be claimed.
- Passing a turn without claiming is not allowed.
- The goal is to avoid forming a polygon or cycle with a single color. Note that only polygons with vertices on the graph vertices count.
- The game ends when such a polygon is formed.
- The player who formed the polygon loses. The other player scores a point.
- If all edges have been claimed but no single-color polygon has been formed, the game ends in a tie. Both players score a point.
- Points accumulate through rounds.
- Rounds can be played on the same board or on different boards if available.
- The player who scored more points after all rounds is the winner.

See the following examples for clarification and further insight.



In this game the red player lost when they formed the polygon filled on red. Blue scores a point. Note that the blue triangle on the bottom right does not count as a cycle since one of its vertices is not a graph node.



On this second example, blue has lost. The four blue edges form a polygon. While this is a non-convex polygon, there are no restrictions about that. This is still a cycle since starting on any node you can walk all the blue lines and end back at the starting node.

Procedures (20 minutes)

1. Explain the rules. Consider playing a demonstration game on the whiteboard to make sure the rules were understood.
2. Organize the class into pairs and distribute the materials. Start with the hexagonal graph ($K_{6,6}$).
3. Have them play a practice round and go around the room verifying that they are following the rules correctly.
4. After playing two or three rounds on this graph ($K_{6,6}$), pause and ask a couple of students to draw on the board examples where they were able to color many edges without forming polygons. If other students have examples with even more edges, allow them to show them. The goal is to work as a class to find the largest possible set of edges that does not contain a cycle.
5. Allow more play time with other boards, encouraging students to think about the previous question (about the largest possible set of edges that does not contain a cycle) in their new board before starting to play.
6. You can also suggest that students design their own boards. Any graph works!

Closure (10 minutes)

Close with a discussion about the activity. Allow students to ask any questions they might have about the game, graphs, matroids, or Carolyn Mahoney.

Explain that the winning colorings, the ones that do not contain polygons or cycles, are called "independent sets" in the theory of matroids that Mahoney studied. Clarify that this is an important concept that mathematicians are still studying and that, in fact, a mathematician was recently awarded a very important medal for his work in this area.

You might also want to touch on the point that Carolyn Mahoney wanted to improve education in areas where educators and students alike were struggling with understanding math. You can organize a discussion around the question: why is it important to make education available to everyone?

References

- Mahoney, Carolyn Ray Boone. On the unimodality of the independent set numbers of a class of matroids. Dissertation presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Graduate School of The Ohio State University. 1982
- Mahoney, Carolyn R. Mathematics Education in Rural Communities: A Mathematician's View. Working Paper Series. Appalachian Collaborative Center for Learning, Assessment and Instruction in Mathematics. Ohio University. March, 2003.
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- Wang, Charles. What is a matroid? Available on https://math.berkeley.edu/~charles/whatis/matroids.pdf